Appendix C: Advanced Relational Database Design

- Reasoning with MVDs
- Higher normal forms
  - Join dependencies and PJNF
  - DKNF
Theory of Multivalued Dependencies

Let $D$ denote a set of functional and multivalued dependencies. The closure $D^+$ of $D$ is the set of all functional and multivalued dependencies logically implied by $D$.

Sound and complete inference rules for functional and multivalued dependencies:

1. **Reflexivity rule.** If $\alpha$ is a set of attributes and $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ holds.

2. **Augmentation rule.** If $\alpha \rightarrow \beta$ holds and $\gamma$ is a set of attributes, then $\gamma \alpha \rightarrow \gamma \beta$ holds.

3. **Transitivity rule.** If $\alpha \rightarrow \beta$ holds and $\gamma \alpha \rightarrow \gamma \beta$ holds, then $\alpha \rightarrow \gamma$ holds.
4. **Complementation rule.** If $\alpha \rightarrow \beta$ holds, then $\alpha \rightarrow R - \beta - \alpha$ holds.

5. **Multivalued augmentation rule.** If $\alpha \rightarrow \beta$ holds and $\gamma \subseteq R$ and $\delta \subseteq \gamma$, then $\gamma \alpha \delta \beta$ holds.

6. **Multivalued transitivity rule.** If $\alpha \rightarrow \beta$ holds and $\beta \rightarrow \gamma$ holds, then $\alpha \rightarrow \gamma - \beta$ holds.

7. **Replication rule.** If $\alpha \rightarrow \beta$ holds, then $\alpha \rightarrow \beta$.

8. **Coalescence rule.** If $\alpha \rightarrow \beta$ holds and $\gamma \subseteq \beta$ and there is a $\delta$ such that $\delta \subseteq R$ and $\delta \cap \beta = \emptyset$ and $\delta \gamma$, then $\alpha \gamma$ holds.
We can simplify the computation of the closure of $D$ by using the following rules (proved using rules 1-8).

- **Multivalued union rule.** If $\alpha \rightarrow\rightarrow \beta$ holds and $\alpha \rightarrow\rightarrow \gamma$ holds, then $\alpha \rightarrow\rightarrow \beta \gamma$ holds.

- **Intersection rule.** If $\alpha \rightarrow\rightarrow \beta$ holds and $\alpha \rightarrow\rightarrow \gamma$ holds, then $\alpha \rightarrow\rightarrow \beta \cap \gamma$ holds.

- **Difference rule.** If $\alpha \rightarrow\rightarrow \beta$ holds and $\alpha \rightarrow\rightarrow \gamma$ holds, then $\alpha \rightarrow\rightarrow \beta - \gamma$ holds and $\alpha \rightarrow\rightarrow \gamma - \beta$ holds.
Example

- $D = \{A \rightarrow B, B \rightarrow HI, CG \rightarrow H\}$

Some members of $D^+$:

- $A \rightarrow CGHI$.
  Since $A \rightarrow B$, the complementation rule (4) implies that $A \rightarrow R - B - A$.
  Since $R - B - A = CGHI$, so $A \rightarrow CGHI$.

- $A \rightarrow HI$.
  Since $A \rightarrow B$ and $B \rightarrow HI$, the multivalued transitivity rule (6) implies that $B \rightarrow HI - B$.
  Since $HI - B = HI$, $A \rightarrow HI$. 
Example (Cont.)

- Some members of $D^+$ (cont.):
  - $B \Rightarrow H$.
    Apply the coalescence rule (8); $B \Rightarrow HI$ holds.
    Since $H \subseteq HI$ and $CG \Rightarrow H$ and $CG \cap HI = \emptyset$, the
    coalescence rule is satisfied with $\alpha$ being $B$, $\beta$ being $HI$, $\delta$ being
    $CG$, and $\gamma$ being $H$. We conclude that $B \Rightarrow H$.
  - $A \Rightarrow CG$.
    $A \Rightarrow CGHI$ and $A \Rightarrow HI$.
    By the difference rule, $A \Rightarrow CGHI \setminus HI$.
    Since $CGHI \setminus HI = CG$, $A \Rightarrow CG$. 
Normalization Using Join Dependencies

Join dependencies constrain the set of legal relations over a schema $R$ to those relations for which a given decomposition is a lossless-join decomposition.

Let $R$ be a relation schema and $R_1, R_2, ..., R_n$ be a decomposition of $R$. If $R = R_1 \cup R_2 \cup ... \cup R_n$, we say that a relation $r(R)$ satisfies the join dependency $*(R_1, R_2, ..., R_n)$ if:

$$ r = \prod_{R_1}(r) \bowtie \prod_{R_2}(r) \bowtie ... \bowtie \prod_{R_n}(r) $$

A join dependency is trivial if one of the $R_i$ is $R$ itself.

A join dependency $*(R_1, R_2)$ is equivalent to the multivalued dependency $R_1 \cap R_2 \rightarrow R_2$. Conversely, $\alpha \rightarrow_{\beta} \beta$ is equivalent to $*(\alpha \cup (R - \beta), \alpha \cup \beta)$

However, there are join dependencies that are not equivalent to any multivalued dependency.
Project-Join Normal Form (PJNF)

A relation schema $R$ is in PJNF with respect to a set $D$ of functional, multivalued, and join dependencies if for all join dependencies in $D^+$ of the form

$$(R_1, R_2, \ldots, R_n)$$

where each $R_i \subseteq R$

and $R = R_1 \cup R_2 \cup \ldots \cup R_n$

at least one of the following holds:

- $$(R_1, R_2, \ldots, R_n)$$ is a trivial join dependency.
- Every $R_i$ is a superkey for $R$.

Since every multivalued dependency is also a join dependency, every PJNF schema is also in 4NF.
Example

- Consider Loan-info-schema = (branch-name, customer-name, loan-number, amount).
- Each loan has one or more customers, is in one or more branches and has a loan amount; these relationships are independent, hence we have the join dependency
- *(=(loan-number, branch-name), (loan-number, customer-name), (loan-number, amount))
- Loan-info-schema is not in PJNF with respect to the set of dependencies containing the above join dependency. To put Loan-info-schema into PJNF, we must decompose it into the three schemas specified by the join dependency:
  - (loan-number, branch-name)
  - (loan-number, customer-name)
  - (loan-number, amount)
Domain-Key Normal Form (DKNY)

- **Domain declaration.** Let $A$ be an attribute, and let $\text{dom}$ be a set of values. The domain declaration $A \subseteq \text{dom}$ requires that the $A$ value of all tuples be values in $\text{dom}$.

- **Key declaration.** Let $R$ be a relation schema with $K \subseteq R$. The key declaration $\text{key}(K)$ requires that $K$ be a superkey for schema $R$ ($K \rightarrow R$). All key declarations are functional dependencies but not all functional dependencies are key declarations.

- **General constraint.** A general constraint is a predicate on the set of all relations on a given schema.

- Let $D$ be a set of domain constraints and let $K$ be a set of key constraints for a relation schema $R$. Let $G$ denote the general constraints for $R$. Schema $R$ is in DKNF if $D \cup K$ logically imply $G$. 
Example

- Accounts whose `account-number` begins with the digit 9 are special high-interest accounts with a minimum balance of 2500.
- General constraint: ``If the first digit of \( t[account-number] \) is 9, then \( t[balance] \geq 2500.``
- DKNF design:
  
  \[
  \text{Regular-acct-schema} = (\text{branch-name}, \text{account-number}, \text{balance})
  \]
  
  \[
  \text{Special-acct-schema} = (\text{branch-name}, \text{account-number}, \text{balance})
  \]
  
- Domain constraints for \{`Special-acct-schema`\} require that for each account:
  
  - The account number begins with 9.
  - The balance is greater than 2500.
Let $R = (A_1, A_2, ..., A_n)$ be a relation schema. Let $\text{dom}(A_i)$ denote the domain of attribute $A_i$, and let all these domains be infinite. Then all domain constraints $D$ are of the form $A_i \subseteq \text{dom}(A_i)$.

Let the general constraints be a set $G$ of functional, multivalued, or join dependencies. If $F$ is the set of functional dependencies in $G$, let the set $K$ of key constraints be those nontrivial functional dependencies in $F^+$ of the form $\alpha \rightarrow R$.

Schema $R$ is in PJNF if and only if it is in DKNF with respect to $D$, $K$, and $G$. 