Appendix C: Other Relational Languages
Appendix C: Other Relational Languages

- Query-by-Example (QBE)
- Access
- Datalog
Query-by-Example (QBE)

- Basic Structure
- Queries on One Relation
- Queries on Several Relations
- The Condition Box
- The Result Relation
- Ordering the Display of Tuples
- Aggregate Operations
- Modification of the Database
QBE — Basic Structure

- A graphical query language which is based (roughly) on the domain relational calculus
- **Two dimensional syntax** – system creates templates of relations that are requested by users
- Queries are expressed “by example”
### QBE Skeleton Tables for the Bank Example

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>customer</th>
<th>customer_name</th>
<th>customer_street</th>
<th>customer_city</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
QBE Skeleton Tables (Cont.)

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>customer</th>
<th>customer_name</th>
<th>customer_street</th>
<th>customer_city</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
</table>
Find all loan numbers at the Perryridge branch.

- \( x \) is a variable (optional; can be omitted in above query)
- P. means print (display)
- duplicates are removed by default
- To retain duplicates use P.ALL

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. ( x )</td>
<td>Perryridge</td>
<td>P. ALL.</td>
<td>Perryridge</td>
</tr>
</tbody>
</table>
Queries on One Relation (Cont.)

- Display full details of all loans

  - Method 1:

    | loan | loan_number | branch_name | amount |
    |------|-------------|-------------|--------|
    | P. x | P. y        | P. z        |        |

  - Method 2: Shorthand notation

    | loan | loan_number | branch_name | amount |
    |------|-------------|-------------|--------|
    | P.   |             |             |        |
Queries on One Relation (Cont.)

- Find the loan number of all loans with a loan amount of more than $700

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P.</td>
<td>&gt;700</td>
</tr>
</tbody>
</table>

- Find names of all branches that are not located in Brooklyn

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.</td>
<td></td>
<td>¬ Brooklyn</td>
<td></td>
</tr>
</tbody>
</table>
Queries on One Relation (Cont.)

- Find the loan numbers of all loans made jointly to Smith and Jones.

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Smith</td>
<td>P._x</td>
</tr>
<tr>
<td></td>
<td>Jones</td>
<td>_x</td>
</tr>
</tbody>
</table>

- Find all customers who live in the same city as Jones.

<table>
<thead>
<tr>
<th>customer</th>
<th>customer_name</th>
<th>customer_street</th>
<th>customer_city</th>
</tr>
</thead>
<tbody>
<tr>
<td>P._x</td>
<td>Jones</td>
<td></td>
<td>_y</td>
</tr>
</tbody>
</table>
Find the names of all customers who have a loan from the Perryridge branch.

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>_x</td>
<td></td>
<td>Perryridge</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P._y</td>
<td></td>
<td>_x</td>
</tr>
</tbody>
</table>
Queries on Several Relations (Cont.)

- Find the names of all customers who have both an account and a loan at the bank.

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P._x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>_x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Negation in QBE

- Find the names of all customers who have an account at the bank, but do not have a loan from the bank.

\[
\begin{array}{ccc}
\text{depositor} & \text{customer\_name} & \text{account\_number} \\
\hline
P.\_x \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{borrower} & \text{customer\_name} & \text{loan\_number} \\
\hline
\neg & x \\
\end{array}
\]

\(\neg\) means “there does not exist”
### Negation in QBE (Cont.)

- Find all customers who have at least two accounts.

```
<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P._x</td>
<td>_x</td>
<td>_y</td>
</tr>
<tr>
<td>_x</td>
<td></td>
<td>¬ _y</td>
</tr>
</tbody>
</table>
```

¬ means “not equal to”
The Condition Box

- Allows the expression of constraints on domain variables that are either inconvenient or impossible to express within the skeleton tables.
- Complex conditions can be used in condition boxes
- Example: Find the loan numbers of all loans made to Smith, to Jones, or to both jointly

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>_n</td>
<td>P._x</td>
<td></td>
</tr>
</tbody>
</table>

conditions

\[ _n = \text{Smith or } _n = \text{Jones} \]
QBE supports an interesting syntax for expressing alternative values.

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.</td>
<td>$_x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**conditions**

$\_x = \text{(Brooklyn or Queens)}$
Find all account numbers with a balance greater than $1,300 and less than $1,500.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P.</td>
<td></td>
<td>_x</td>
</tr>
</tbody>
</table>

conditions

_x ≥ 1300
_x ≤ 1500

Find all account numbers with a balance greater than $1,300 and less than $2,000 but not exactly $1,500.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P.</td>
<td></td>
<td>_x</td>
</tr>
</tbody>
</table>

conditions

_x = ( ≥ 1300 \text{ and} ≤ 2000 \text{ and} \neg 1500)
Find all branches that have assets greater than those of at least one branch located in Brooklyn.

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>P._x</td>
<td></td>
<td>Brooklyn</td>
<td>_y</td>
</tr>
</tbody>
</table>

\[ _y > _z \]
Find the `customer_name`, `account_number`, and `balance` for all customers who have an account at the Perryridge branch.

- We need to:
  - Join `depositor` and `account`.
  - Project `customer_name`, `account_number` and `balance`.

- To accomplish this we:
  - Create a skeleton table, called `result`, with attributes `customer_name`, `account_number`, and `balance`.
  - Write the query.
The resulting query is:

```
+----------------+----------------+----------------+----------------+
<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-y</td>
<td>Perryridge</td>
<td>-z</td>
</tr>
</tbody>
</table>

+----------------+----------------+----------------+----------------+
<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-x</td>
<td>-y</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+----------------+----------------+----------------+----------------+
<table>
<thead>
<tr>
<th>result</th>
<th>customer_name</th>
<th>account_number</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.</td>
<td>-x</td>
<td>-y</td>
<td>-z</td>
</tr>
</tbody>
</table>
```
Ordering the Display of Tuples

- **AO** = ascending order; **DO** = descending order.
- Example: list in ascending alphabetical order all customers who have an account at the bank.

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P.AO.</td>
<td></td>
</tr>
</tbody>
</table>

- When sorting on multiple attributes, the sorting order is specified by including with each sort operator (AO or DO) an integer surrounded by parentheses.
- Example: List all account numbers at the Perryridge branch in ascending alphabetic order with their respective account balances in descending order.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P.AO(1).</td>
<td>Perryridge</td>
<td>P.DO(2).</td>
</tr>
</tbody>
</table>
Aggregate Operations

- The aggregate operators are AVG, MAX, MIN, SUM, and CNT
- The above operators must be postfixed with “ALL” (e.g., SUM.ALL. or AVG.ALL._x) to ensure that duplicates are not eliminated.
- Example: Find the total balance of all the accounts maintained at the Perryridge branch.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Perryridge</td>
<td>P.SUM.ALL.</td>
</tr>
</tbody>
</table>
Aggregate Operations (Cont.)

- UNQ is used to specify that we want to eliminate duplicates.
- Find the total number of customers having an account at the bank.

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.CNT.UNQ.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Find the average balance at each branch.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P.G.</td>
<td>P.AVG.ALL..x</td>
</tr>
</tbody>
</table>

The “G” in “P.G” is analogous to SQL’s `group by` construct.

The “ALL” in the “P.AVG.ALL” entry in the `balance` column ensures that all balances are considered.

To find the average account balance at only those branches where the average account balance is more than $1,200, we simply add the condition box:

```
conditions

AVG.ALL..x > 1200
```
Query Example

- Find all customers who have an account at all branches located in Brooklyn.
  - Approach: for each customer, find the number of branches in Brooklyn at which they have accounts, and compare with total number of branches in Brooklyn.
  - QBE does not provide subquery functionality, so both above tasks have to be combined in a single query.
    - Can be done for this query, but there are queries that require subqueries and cannot always be expressed in QBE.

- In the query on the next page
  - CNT.UNQ.ALL._w specifies the number of distinct branches in Brooklyn. Note: The variable _w is not connected to other variables in the query.
  - CNT.UNQ.ALL._z specifies the number of distinct branches in Brooklyn at which customer x has an account.
### Query Example (Cont.)

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.G._x</td>
<td></td>
<td>_y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>_y</td>
<td></td>
<td>_z</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>_z</td>
<td></td>
<td>Brooklyn</td>
<td></td>
</tr>
<tr>
<td>_w</td>
<td></td>
<td>Brooklyn</td>
<td></td>
</tr>
</tbody>
</table>

### Conditions

\[
\text{CNT.UNQ}._z = \\
\text{CNT.UNQ}._w
\]
Modification of the Database – Deletion

- Deletion of tuples from a relation is expressed by use of a D. command. In the case where we delete information in only some of the columns, null values, specified by –, are inserted.

- Delete customer Smith

<table>
<thead>
<tr>
<th>customer</th>
<th>customer_name</th>
<th>customer_street</th>
<th>customer_city</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.</td>
<td>Smith</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Delete the branch_city value of the branch whose name is “Perryridge”.

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perryridge</td>
<td>D.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Deletion Query Examples

- Delete all loans with a loan amount greater than $1300 and less than $1500.
  - For consistency, we have to delete information from loan and borrower tables

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.</td>
<td>-y</td>
<td>_x</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.</td>
<td></td>
<td>-y</td>
</tr>
</tbody>
</table>

**conditions**

\[-x = (\geq 1300 \textit{ and } \leq 1500)\]
Delete all accounts at branches located in Brooklyn.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>_y</td>
<td>_x</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.</td>
<td></td>
<td>_y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>_x</td>
<td></td>
<td>Brooklyn</td>
<td></td>
</tr>
</tbody>
</table>
Modification of the Database – Insertion

- Insertion is done by placing the I. operator in the query expression.

- Insert the fact that account A-9732 at the Perryridge branch has a balance of $700.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>A-9732</td>
<td>Perryridge</td>
<td>700</td>
</tr>
</tbody>
</table>
Provide as a gift for all loan customers of the Perryridge branch, a new $200 savings account for every loan account they have, with the loan number serving as the account number for the new savings account.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>_x</td>
<td>Perryridge</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>depositor</th>
<th>customer_name</th>
<th>account_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>_y</td>
<td>_x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>loan</th>
<th>loan_number</th>
<th>branch_name</th>
<th>amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>_x</td>
<td>Perryridge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>borrower</th>
<th>customer_name</th>
<th>loan_number</th>
</tr>
</thead>
<tbody>
<tr>
<td>_y</td>
<td>_x</td>
<td></td>
</tr>
</tbody>
</table>
Use the U. operator to change a value in a tuple without changing all values in the tuple. QBE does not allow users to update the primary key fields.

Update the asset value of the Perryridge branch to $10,000,000.

<table>
<thead>
<tr>
<th>branch</th>
<th>branch_name</th>
<th>branch_city</th>
<th>assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Perryridge</td>
<td></td>
<td>U.10000000</td>
</tr>
</tbody>
</table>

Increase all balances by 5 percent.

<table>
<thead>
<tr>
<th>account</th>
<th>account_number</th>
<th>branch_name</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>U._x * 1.05</td>
</tr>
</tbody>
</table>
Microsoft Access supports a variant of QBE called Graphical Query By Example (GQBE).

GQBE differs from QBE in the following ways:

- Attributes of relations are listed vertically, one below the other, instead of horizontally.
- Instead of using variables, lines (links) between attributes are used to specify that their values should be the same.
  - Links are added automatically on the basis of attribute name, and the user can then add or delete links.
  - By default, a link specifies an inner join, but can be modified to specify outer joins.
- Conditions, values to be printed, as well as group by attributes are all specified together in a box called the design grid.
Example query: Find the *customer_name*, *account_number* and *balance* for all accounts at the Perryridge branch.
An Aggregation Query in Access QBE

- Find the *name*, *street* and *city* of all customers who have more than one account at the bank.
The row labeled **Total** specifies:

- which attributes are group by attributes.
- which attributes are to be aggregated upon (and the aggregate function).
- For attributes that are neither group by nor aggregated, we can still specify conditions by selecting **where** in the Total row and listing the conditions below.

As in SQL, if group by is used, only group by attributes and aggregate results can be output.
Datalog

- Basic Structure
- Syntax of Datalog Rules
- Semantics of Nonrecursive Datalog
- Safety
- Relational Operations in Datalog
- Recursion in Datalog
- The Power of Recursion
Basic Structure

- Prolog-like logic-based language that allows recursive queries; based on first-order logic.
- A Datalog program consists of a set of rules that define views.
- Example: define a view relation v1 containing account numbers and balances for accounts at the Perryridge branch with a balance of over $700.

\[ v1(A, B) :– \text{account}(A, \text{“Perryridge”}, B), B > 700. \]

- Retrieve the balance of account number “A-217” in the view relation v1.

\[ ? \ v1(\text{“A-217”}, B). \]

- To find account number and balance of all accounts in v1 that have a balance greater than 800

\[ ? \ v1(A,B), B > 800 \]
Example Queries

- Each rule defines a set of tuples that a view relation must contain.
  - E.g., $v1(A, B) :\leftarrow account(A, \text{"Perryridge"}, B), B > 700$
    is read as:
      
      for all $A, B$
      
      if $(A, \text{"Perryridge"}, B) \in account \text{ and } B > 700$
      
      then $(A, B) \in v1$

- The set of tuples in a view relation is then defined as the union of all
  the sets of tuples defined by the rules for the view relation.

- Example:
  
  $interest_rate(A, 5) :\leftarrow account(A, N, B), B < 10000$
  
  $interest_rate(A, 6) :\leftarrow account(A, N, B), B >= 10000$
Negation in Datalog

- Define a view relation \( c \) that contains the names of all customers who have a deposit but no loan at the bank:

  \[
  c(N) :\neg \text{depositor}(N, A), \textbf{not} \: \text{is_borrower}(N).
  \]

  \[
  \text{is_borrower}(N) :\neg \text{borrower}(N,L).
  \]

- NOTE: using \textbf{not} \( \text{borrower}(N, L) \) in the first rule results in a different meaning, namely there is some loan \( L \) for which \( N \) is not a borrower.

  - To prevent such confusion, we require all variables in negated “predicate” to also be present in non-negated predicates.
Named Attribute Notation

- Datalog rules use a positional notation that is convenient for relations with a small number of attributes.
- It is easy to extend Datalog to support named attributes.
  - E.g., \( v1 \) can be defined using named attributes as
    \[
    v1 \,(\text{account\_number} \, A, \, \text{balance} \, B) \,:– \\
    \quad \text{account} \,(\text{account\_number} \, A, \, \text{branch\_name} \, \text{“Perryridge”}, \, \\
    \, \text{balance} \, B), \\
    \, B > 700.
    \]
We formally define the syntax and semantics (meaning) of Datalog programs, in the following steps:

1. We define the syntax of predicates, and then the syntax of rules.
2. We define the semantics of individual rules.
3. We define the semantics of non-recursive programs, based on a layering of rules.
4. It is possible to write rules that can generate an infinite number of tuples in the view relation. To prevent this, we define what rules are “safe”. Non-recursive programs containing only safe rules can only generate a finite number of answers.
5. It is possible to write recursive programs whose meaning is unclear. We define what recursive programs are acceptable, and define their meaning.
Syntax of Datalog Rules

- A **positive literal** has the form
  \[ p(t_1, t_2, \ldots, t_n) \]
  - \( p \) is the name of a relation with \( n \) attributes
  - each \( t_i \) is either a constant or variable

- A **negative literal** has the form
  \[ \text{not } p(t_1, t_2, \ldots, t_n) \]

- Comparison operations are treated as positive predicates
  - E.g., \( X > Y \) is treated as a predicate \( >(X,Y) \)
  - “>” is conceptually an (infinite) relation that contains all pairs of values such that the first value is greater than the second value

- Arithmetic operations are also treated as predicates
  - E.g., \( A = B + C \) is treated as \( +(B, C, A) \), where the relation “+” contains all triples such that the third value is the sum of the first two
Syntax of Datalog Rules (Cont.)

- **Rules** are built out of literals and have the form:
  \[ p(t_1, t_2, \ldots, t_n) : L_1, L_2, \ldots, L_m. \]
  
  - **head**
  - **body**
  
  - each \( L_i \) is a literal
  - head – the literal \( p(t_1, t_2, \ldots, t_n) \)
  - body – the rest of the literals

- A **fact** is a rule with an empty body, written in the form:
  \[ p(v_1, v_2, \ldots, v_n). \]
  
  - indicates tuple \((v_1, v_2, \ldots, v_n)\) is in relation \( p \)

- A **Datalog program** is a set of rules.
Semantics of a Rule

- A **ground instantiation of a rule** (or simply **instantiation**) is the result of replacing each variable in the rule by some constant.
  
  - E.g., Rule defining $v1$
    
    $$v1(A,B) :- \text{account (A,\text{"Perryridge"}, B ), } B > 700.$$  

  - An instantiation above rule:
    
    $$v1 (\text{"A-217", 750}) :- \text{account ("A-217", \text{"Perryridge"}, 750), } 750 > 700.$$  

- The body of rule instantiation $R'$ is **satisfied** in a set of facts (database instance) $I$ if
  
  1. For each positive literal $q_i(v_{i,1}, \ldots, v_{i,n_i})$ in the body of $R'$, $I$ contains the fact $q_i(v_{i,1}, \ldots, v_{i,n_i})$.
  
  2. For each negative literal **not** $q_j(v_{j,1}, \ldots, v_{j,n_j})$ in the body of $R'$, $I$ does not contain the fact $q_j(v_{j,1}, \ldots, v_{j,n_j})$. 
Semantics of a Rule (Cont.)

- We define the set of facts that can be inferred from a given set of facts \( l \) using rule \( R \) as:
  \[
  \text{infer}(R, l) = \{ p(t_1, ..., t_n) \mid \text{there is a ground instantiation } R' \text{ of } R \\
  \text{where } p(t_1, ..., t_n) \text{ is the head of } R', \text{ and} \\
  \text{the body of } R' \text{ is satisfied in } l \}
  \]

- Given a set of rules \( \mathcal{R} = \{R_1, R_2, ..., R_n\} \), we define
  \[
  \text{infer}(\mathcal{R}, l) = \text{infer}(R_1, l) \cup \text{infer}(R_2, l) \cup ... \cup \text{infer}(R_n, l)
  \]
Layering of Rules

- Define the interest on each account in Perryridge

\[
\text{interest}(A, I) :- \text{perryridge_account}(A, B), \\
\text{interest_rate}(A, R), I = B * R/100.
\]

\[
\text{perryridge_account}(A, B) :- \text{account}(A, \text{“Perryridge”}, B).
\]

\[
\text{interest_rate}(A, 5) :- \text{account}(N, A, B), B < 10000.
\]

\[
\text{interest_rate}(A, 6) :- \text{account}(N, A, B), B \geq 10000.
\]

- Layering of the view relations
Layering Rules (Cont.)

Formally:

- A relation is a layer 1 if all relations used in the bodies of rules defining it are stored in the database.
- A relation is a layer 2 if all relations used in the bodies of rules defining it are either stored in the database, or are in layer 1.
- A relation $p$ is in layer $i + 1$ if
  - it is not in layers 1, 2, ..., $i$
  - all relations used in the bodies of rules defining a $p$ are either stored in the database, or are in layers 1, 2, ..., $i$
Let the layers in a given program be 1, 2, ..., n. Let $R_i$ denote the set of all rules defining view relations in layer $i$.

- Define $I_0 =$ set of facts stored in the database.
- Recursively define $I_{i+1} = I_i \cup \text{infer} (R_{i+1}, I_i)$
- The set of facts in the view relations defined by the program (also called the semantics of the program) is given by the set of facts $I_n$ corresponding to the highest layer $n$.

Note: Can instead define semantics using view expansion like in relational algebra, but above definition is better for handling extensions such as recursion.
Safety

- It is possible to write rules that generate an infinite number of answers.

\[
gt(X, Y) :- X > Y
\]
\[
not\_in\_loan (B, L) :- \text{not} \ enthusiast (B, L)
\]

To avoid this possibility Datalog rules must satisfy the following conditions.

- Every variable that appears in the head of the rule also appears in a non-arithmetic positive literal in the body of the rule.
  - This condition can be weakened in special cases based on the semantics of arithmetic predicates, for example to permit the rule
    \[
p (A) :- q (B), A = B + 1
\]
- Every variable appearing in a negative literal in the body of the rule also appears in some positive literal in the body of the rule.
Relational Operations in Datalog

■ Project out attribute `account_name` from account.

\[
\text{query} (A) :- \text{account} (A, N, B).
\]

■ Cartesian product of relations \( r_1 \) and \( r_2 \).

\[
\text{query} (X_1, X_2, ..., X_n, Y_1, Y_1, Y_2, ..., Y_m) :- \\
\quad r_1 (X_1, X_2, ..., X_n), r_2 (Y_1, Y_2, ..., Y_m).
\]

■ Union of relations \( r_1 \) and \( r_2 \).

\[
\text{query} (X_1, X_2, ..., X_n) :- r_1 (X_1, X_2, ..., X_n), \\
\text{query} (X_1, X_2, ..., X_n) :- r_2 (X_1, X_2, ..., X_n),
\]

■ Set difference of \( r_1 \) and \( r_2 \).

\[
\text{query} (X_1, X_2, ..., X_n) :- r_1 (X_1, X_2, ..., X_n), \\
\quad \text{not} \ r_2 (X_1, X_2, ..., X_n),
\]
Recursion in Datalog

- Suppose we are given a relation

  \[ \text{manager} \ (X, \ Y) \]

  containing pairs of names X, Y such that Y is a manager of X (or equivalently, X is a direct employee of Y).

- Each manager may have direct employees, as well as indirect employees.
  - Indirect employees of a manager, say Jones, are employees of people who are direct employees of Jones, or recursively, employees of people who are indirect employees of Jones.

- Suppose we wish to find all (direct and indirect) employees of manager Jones. We can write a recursive Datalog program.

  \[
  \text{empl\_jones} \ (X) :\ :- \ \text{manager} \ (X, \ Jones).
  \]

  \[
  \text{empl\_jones} \ (X) :\ :- \ \text{manager} \ (X, \ Y), \ \text{empl\_jones} \ (Y).
  \]
Semantics of Recursion in Datalog

- Assumption (for now): program contains no negative literals
- The view relations of a recursive program containing a set of rules $\mathcal{R}$ are defined to contain exactly the set of facts $l$ computed by the iterative procedure $Datalog$-Fixpoint

```plaintext
procedure Datalog-Fixpoint
  $l = $ set of facts in the database
  repeat
    Old$_l = l$
    $l = l \cup infer (\mathcal{R}, l)$
  until $l = Old_l$
```

- At the end of the procedure, $infer (\mathcal{R}, l) \subseteq l$
  - $Infer (\mathcal{R}, l) = l$ if we consider the database to be a set of facts that are part of the program
- $l$ is called a **fixed point** of the program.
Example of Datalog-FixPoint Iteration

<table>
<thead>
<tr>
<th>employee_name</th>
<th>manager_name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alon</td>
<td>Barinsky</td>
</tr>
<tr>
<td>Barinsky</td>
<td>Estovar</td>
</tr>
<tr>
<td>Corbin</td>
<td>Duarte</td>
</tr>
<tr>
<td>Duarte</td>
<td>Jones</td>
</tr>
<tr>
<td>Estovar</td>
<td>Jones</td>
</tr>
<tr>
<td>Jones</td>
<td>Klinger</td>
</tr>
<tr>
<td>Rensal</td>
<td>Klinger</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>Tuples in emp1_jones</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(Duarte), (Estovar)</td>
</tr>
<tr>
<td>2</td>
<td>(Duarte), (Estovar), (Barinsky), (Corbin)</td>
</tr>
<tr>
<td>3</td>
<td>(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)</td>
</tr>
<tr>
<td>4</td>
<td>(Duarte), (Estovar), (Barinsky), (Corbin), (Alon)</td>
</tr>
</tbody>
</table>
A More General View

- Create a view relation `empl` that contains every tuple \((X, Y)\) such that \(X\) is directly or indirectly managed by \(Y\).

\[
\begin{align*}
\text{empl} (X, Y) & :\neg \text{manager} (X, Y). \\
\text{empl} (X, Y) & :\neg \text{manager} (X, Z), \text{empl} (Z, Y)
\end{align*}
\]

- Find the direct and indirect employees of Jones.

\[
? \text{empl} (X, "Jones").
\]

- Can define the view `empl` in another way too:

\[
\begin{align*}
\text{empl} (X, Y) & :\neg \text{manager} (X, Y). \\
\text{empl} (X, Y) & :\neg \text{empl} (X, Z), \text{manager} (Z, Y).
\end{align*}
\]
The Power of Recursion

- Recursive views make it possible to write queries, such as transitive closure queries, that cannot be written without recursion or iteration.
  - Intuition: Without recursion, a non-recursive non-iterative program can perform only a fixed number of joins of manager with itself.
    - This can give only a fixed number of levels of managers.
    - Given a program we can construct a database with a greater number of levels of managers on which the program will not work.
Recursion in SQL

- Starting with SQL:1999, SQL permits recursive view definition
- E.g., query to find all employee-manager pairs

```sql
WITH recursive empl (emp, mgr) AS (
    SELECT emp, mgr
    FROM manager
    UNION
    SELECT manager.emp, empl.mgr
    FROM manager, empl
    WHERE manager.mgr = empl.emp
)
SELECT *
FROM empl
```
Monotonicity

- A view $V$ is said to be **monotonic** if given any two sets of facts $I_1$ and $I_2$ such that $I_1 \subseteq I_2$, then $E_v(I_1) \subseteq E_v(I_2)$, where $E_v$ is the expression used to define $V$.

- A set of rules $R$ is said to be monotonic if $I_1 \subseteq I_2$ implies $\text{infer}(R, I_1) \subseteq \text{infer}(R, I_2)$.

- Relational algebra views defined using only the operations: $\Pi$, $\sigma$, $\times$, $\cup$, $|X|$, $\cap$, and $\rho$ (as well as operations like natural join defined in terms of these operations) are monotonic.

- Relational algebra views defined using set difference ($-$) may not be monotonic.

- Similarly, Datalog programs without negation are monotonic, but Datalog programs with negation may not be monotonic.
Non-Monotonicity

Procedure *Datalog-Fixpoint* is sound provided the rules in the program are monotonic.

- Otherwise, it may make some inferences in an iteration that cannot be made in a later iteration. E.g., given the rules
  
  \[
  a :\neg b.
  b : c.
  \]

  Then \(a\) can be inferred initially, before \(b\) is inferred, but not later.

- We can extend the procedure to handle negation so long as the program is “stratified”: intuitively, so long as negation is not mixed with recursion
Non-Monotonicity (Cont.)

- There are useful queries that cannot be expressed by a stratified program
  - Example: given information about the number of each subpart in each part, in a part-subpart hierarchy, find the total number of subparts of each part.
  - A program to compute the above query would have to mix aggregation with recursion
  - However, so long as the underlying data (part-subpart) has no cycles, it is possible to write a program that mixes aggregation with recursion, yet has a clear meaning
  - There are ways to evaluate some such classes of non-stratified programs